Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 2	DAILY TUTORIAL SHEET 10

186.(AD) We have a total of 4^5 ways. But a scenario has also occurred when all the 5 prizes have gone to a single person. This has occurred 4 times since there are 4 persons. We need to deduct this case. So, number of ways = $4^5 - {}^4C_1 = 4^5 - 4 = 1024 - 4 = 1020$

187.(AB) The total number of attempts that can be made to open the lock is $(15)^3$. Out of these, there is just one attempt in which lock will open.

So, $N = (15)^3 - 1 = (15-1)\left[(15)^2 + 15 + 1\right] = 14(225 + 15 + 1) = 2 \times 7 \times 241$ Clearly, N is divisible by 482 and 2, 7, 241 are prime numbers.

188.(AC) The number of zeroes is decided by the number of 10s appearing which in turn is decided by the minimum of number of 2s and 5s. Exponent of 2 in $\lfloor 70 = \left[\frac{70}{2}\right] + \left[\frac{70}{4}\right] + \left[\frac{70}{8}\right] + \left[\frac{70}{16}\right] + \left[\frac{70}{32}\right] + \left[\frac{70}{64}\right]$

= 35 + 17 + 8 + 4 + 2 + 1 = 67

Exponent of 5 in $\lfloor 70 = \left[\frac{70}{5}\right] + \left[\frac{70}{25}\right] = 14 + 2 = 16$. Cleary '10' will be formed '16' times. So, answer is 16. Number of 6-digit nos. formed using all digits 1, 2, 3, 4, 5, 6 $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$${}^{10}C_2 = \frac{\lfloor 10}{\lfloor 2 \lfloor 8} = \frac{9 \times 10}{2} = 45 \qquad \frac{720}{45} = 16$$

So, option (A) is correct and option (B) is wrong

Number of 4 - digit nos. using 1, 2, 3, 4, 5, 6. $= 6 \times 5 \times 4 \times 3 = 360$

Twice the number $= 2 \times 360 = 720$. So option (C) is correct and (D) is wrong.

189.(A) Suppose the student gets atleast 60% marks of 150 marks in first two papers, then he just gets at most 30 marks in the third paper to make a total of 150 marks.

Let, x_1, x_2 and x_3 be marks obtained in 3 papers respectively. The total marks to be obtained is 150.

Therefore, Sum of marks obtained = 150 $\Rightarrow$ $x_1 + x_2 + x_3 = 150$ **... (i)**

$60 \le x_1 \le 100$; $60 \le x_2 \le 100$; $0 \le x_3 \le 30$.

The required number of ways = No. of integral solutions of **(i)**

= Coefficient of x^{150} in $\{(x^{60} + x^{61} + \ldots + x^{100})^2 (1 + x + x^2 + \ldots + x^{30})\}$

= Coefficient of x^{30} in $\{(1 + x + \ldots + x^{40})^2 (1 + x + \ldots + x^{30})\}$

= Coefficient of x^{30} in $\left(\frac{1 - x^{41}}{1 - x}\right)^2 \left(\frac{1 - x^{31}}{1 - x}\right)$ = Coefficient of x^{30} in $(1 - x)^{-3} = {}^{30 + 3 - 1}C_{3 - 1} = {}^{32}C_2$.

Thus, the student gets atleast 60% marks in first two papers to get 150 marks as total in ${}^{32}C_2$ ways. But the two papers, of atleast 60% marks, can be chosen out of 3 papers in ${}^3C_2$ ways. Hence, the required number of ways = ${}^3C_2 \times {}^{32}C_2$.

190. **(i)** $P_i \cap P_j = \phi \quad i \ne j$.

Every element in X has $(m + 1)$ choices because either it can be selected for P_1 or P_2 or P_3 .... P_m or not get selected in any of the sets. $\Rightarrow$ No. of favorable ways = $(m+1)(m+1) \ldots n$ times $= (m + 1)^n$

(ii) $P_1 \cap P_2 \cap P_3 \ldots\ldots \cap P_m = \phi$.

This means there is no element common to all sets $P_1, P_2, P_3 \ldots. P_m$.

For each element $(a_1, a_2, \ldots a_n)$ there are $(2^m - 1)$ choices to get selected. It can be selected in any sets but not for all sets together, so we subtract 1 from 2^m.

Total ways to select $P_1, P_2, P_3 \ldots P_m$ such that $P_1 \cap P_2 \ldots \cap P_m = \phi = (2^m - 1)^n$.

191.(B) It is possible in two mutually exclusive cases

Case-1: 2 children get none, one child gets three and all remaining 7 children get one each.

Case-2: 2 children get none, 2 children get 2 each and all remaining 6 children get one each. Using formula given in section (6.3), we get:

Case I: Number of ways = $\left(\frac{10!}{3!\,7!}\,\frac{1}{2!}\right)(10!)$; **Case II:** Number of ways = $\left(\frac{10!}{(2!)^2 \cdot 6!}\,\frac{1}{2!}\,\frac{1}{2!}\right)10!$

Thus, total ways = $(10!)^2\left(\frac{1}{3!\,7!\,2!} + \frac{1}{(2!)^4\,6!}\right)$

192.(B) **S₁:** $\underbrace{1\ 2\ 3\ 3}_{\frac{4!}{2!}\text{ ways}}\ \underbrace{4 \text{ or } 5}_{2 \text{ ways}} \times \underbrace{9 - x}_{10 \text{ ways}} \quad \Rightarrow \quad 12 \times 2 \times 10 = 240$ ways.

S₂: Total – both together = ${}^{11}C_5 - {}^{9}C_3 = 378$

S₃: 10 IIT students $T_1, T_2, \ldots T_{10}$ can be arranged in 10! ways. Now the number of ways in which two PET students can be placed will be equal to the number of ways in which 3 consecutive IIT students can be taken i.e., in 8 ways and can be arranged in two ways $\Rightarrow (10!)(8)(2!)$.

Alternatively: 3 IIT students can be selected in ${}^{10}C_3$ ways. Now each selection of 3 IIT and 2 PET students in $P_1\ T_1\ T_2\ T_3\ P_2$ can be arranged in (2!) (3!) ways. Call this box X. Now this X and the remaining IIT students can be arranged in 8! ways. $\Rightarrow$ Total ways ${}^{10}C_3(2!)(3!)(8!)$.

193.(A) Number of positive integral solutions of $\left(\sum_{i=1}^{4} x_i = 6\right) \times \frac{{}^{10}C_1}{4} = \left[\frac{{}^{5}C_3 \times {}^{10}C_1}{4}\right]$

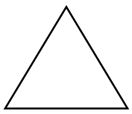
Or alternatively if we consider linear arrangement:

Number of solutions of $\sum_{i=1}^{5} x_i = 6\ [x_1, x_5 \geq 0 \text{ and } x_2, x_3, x_4 \geq 1]$ minus

Number of solutions with $x_1 = x_5 = 0 = \left[{}^{7}C_3 - {}^{5}C_2\right] = 25$

194.(C) 12 boys, 12 girls

$a_1 = 2 \times \lfloor 12 \times \lfloor 12 = 24 \times \lfloor 11 \times \lfloor 12$ and $a_2 = \lfloor 11 \times \lfloor 12$

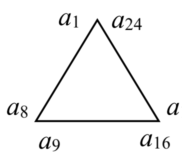
$a_3 =$  $8 \times \lfloor 11 \times \lfloor 12 = \frac{24 \times \lfloor 11 \times \lfloor 12}{3}$

Basically 3 linear permutations give the same permutation around an equilateral triangle.

e.g. Consider the linear permutation. $a_1 a_2, a_8 a_9 \ldots a_{16} a_{17} \ldots a_{24}$

where boys and girls sit alternatively.

Splitting it into 3 sides with 8 persons on each side gives the following permutations around an equilateral Δ:



But note that the linear permutations

$a_9 \ldots a_{16} a_{17} \ldots a_{24}\ a_1 \ldots a_8$

and $a_{17} \ldots a_{24}\ a_1 \ldots a_8\ a_9 \ldots a_{16}$

Also give the same permutation around an equilateral triangle.

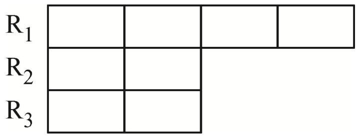
So, number of permutations around an equilateral triangle $= \dfrac{\text{Number of linear permutations}}{3}$

Similarly, number of permutations around a square $= \dfrac{\text{Number of linear permutations}}{4}$

$\Rightarrow \quad a_4 = \dfrac{24 \times \underline{|11} \times \underline{|12}}{4} = 6 \times \underline{|11} \times \underline{|12} \quad \Rightarrow \quad a_2 < a_4 < a_3 < a_1$

195.(B) Fill the boxes without any regards to the condition $= {}^8C_6 \cdot 6!$ out of which there will be only two such cases in which second and third row remain empty. Total ways $= {}^8C_6 \cdot 6! - 2 \cdot 6!$

ALTERNATE SOLUTION:



S. No.	Row-1	Row-2	Row-3
1.	4	1	1
2.	3	2	1
3.	3	1	2
4.	2	2	2

Case I:

Step I: Select 4 elements for Row-1, 1 for Row-2 and one for Row-3 in

${}^6C_2 . {}^2C_1 . {}^1C_1$ ways $= \dfrac{6!}{4!\,1!\,1!}$

Step 2: Arrange 4 selected letters in Row-1 in 4! ways. Select one place in Row-2 in ${}^2C_1$ ways and arrange it in way. Similar way it can be done for Row-3.

No. of ways $= \dfrac{6!}{4!\,1!\,1!} \times \left[4! \times {}^2C_1 \times 1 \times {}^2C_1 \times 1\right] = 6! \times 4$

Case II:

Step I: Select 3 elements for Row-1, 2 for Row-2 and one for Row-3. $\Rightarrow \quad {}^6C_3 . {}^3C_2 . {}^1C_1 = \dfrac{6!}{3!\,2!\,1!}$

Step II: Select 3 places out of 4 for 3 letters in ${}^4C_3$ way and arrange them in 3! ways.

Select 2 places for 2 letters in Row-2 in ${}^2C_2$ ways and arrange them in 2! ways.

Select 1 place for 1 letter in Row-3 in ${}^2C_1$ ways and arrange it in 1! way.

$\Rightarrow \quad ({}^4C_3 \times 3!) \times ({}^2C_2 \times 2!) \times ({}^2C_1 \times 1!) = 4! \times 2! \times 2$

No. of ways $= \dfrac{6!}{3!\,2!\,1!} \times \left(4! \times 2! \times 2!\right) = 6! \times 8$

Case III: Same as case II (Think yourself)

No. of ways = 6! × 8

Case IV:

Step I: Select 2 letters for Row-1, 2 letters for Row-2, 2 letters for Row- 3 in

${}^6C_2 \, {}^4C_2 \, {}^2C_2 = \dfrac{6!}{2!\,2!\,2!}$ ways

Step II: Select 2 places out of 4 places in Row -1 in ${}^4C_2$ ways and arrange them in 2! ways.

Select 2 places for 2 letters in Row-2 in ${}^2C_2$ ways and arrange them in 2! ways.

In similar way, it can be done for Row-3 in 2! ways.

No. of ways $= \left(\dfrac{6!}{2!\,2!\,2!}\right) \times \left({}^4C_2 \times 2! \times 2! \times 2!\right) = 6! \times 6$

Total no. of ways = 6! × 4 + 6! × 8 + 6! × 8 + 6! × 6 = 6! × (4 + 8 + 8 + 6) = 6! × 26

196.(AB) Number of ways $= {}^4C_1 \times$ coefficient of b in $\left(a + \frac{1}{a} + b + \frac{1}{b}\right)^9$

$$\left(a + \frac{1}{a} + b + \frac{1}{b}\right)^9 = \frac{(a+b)^9 (ab+1)^9}{a^9 b^9}$$

Coefficient of $b =$ coefficient of $a^9 b^{10}$ in $(a+b)^9 (1+ab)^9$

General term in $(a+b)^9\ (1+ab)^9$ is ${}^9C_{r_1} a^{9-r_1} b^{r_1}\ {}^9C_{r_2} a^{r_2} b^{r_2}$

Now $9 - r_1 + r_2 = 9$ and $r_1 + r_2 = 10$ given us $r_2 = r_1 = 5$; Hence $4 \times \left({}^9C_5\right)^2 = \left({}^{10}C_5\right)^5$

197.(B) We have: $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ and $x_1 + x_2 + x_3 = 5$

These two equations reduce to $x_4 + x_5 = 15$ **... (i)** and $x_1 + x_2 + x_3 = 5$ **... (ii)**

Since corresponding to each solution of (i) there are solutions of equation (ii). So, total number of solutions of the given system of equations.

= No. of solutions of (i) × No. of solutions of (ii)

$= ({}^{15+2-1}C_1)\ ({}^{5+3-1}C_2) = {}^{16}C_1 \times {}^7C_2 = 336.$

198.(A) One Van out of two available can be selected in ${}^2C_1$ ways.

Out of two possible arrangements (see figure) of adjacent seats, select one in ${}^2C_1$ ways.

Out of remaining 11 seats, select 9 seats in ${}^{11}C_9$ ways.

Arrange 3 girls on 3 seats in 3! ways and 9 boys on 9 seats 9! ways.

So possible arrangement of sitting (for 3 girls and 9 boys in 2 Vans)

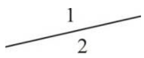
${}^2C_1 \times {}^2C_1 \times {}^{11}C_9 \times 3! \times 9! = 12!$ ways.

199.(C) $n_1 = x_1, x_2, x_3, x_4, x_5$ $n_2 = y_1, y_2, y_3, y_4, y_5$ and n_2 can be added without carrying at any stage if $x_i + y_i \le 9$.

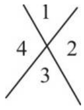
Value of x_5	**Value of y_5**
0	0, 1, 2, ..., 9
1	0, 1, 2, ..., 8
2	0, 1, 2, ..., 7
3	0, 1, 2, 3, 4, 5, 6
4	0, 1, 2, 3, 4, 5
5	0, 1, 2, 3, 4
6	0, 1, 2, 3
7	0, 1, 2
8	0, 1
9	0

Thus, x_5 and y_5 can be selected collectively by $10 + 9 + 8 + ... + 1 = 55$ ways. Similarly, each pair (x_4, y_4), (x_3, y_3) (x_2, y_2), can be selected in 55 ways. But pair (x_1, y_1) can be selected in 1 + 2 + 3 +...+ 8 = 36 ways as in this pair we cannot have 0 or 9. Thus, total number of ways is $36(55)^4$.

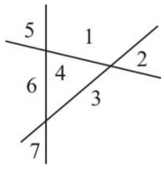
200.(A) **Case I:** If there is only one line: number of sub parts = 2



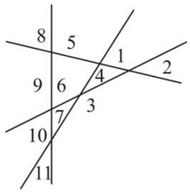
Case 2: If two lines are there: number of sub parts = 2 + 2 = 4



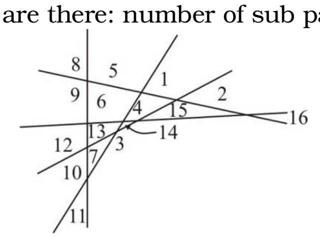
Case 3: If three lines are there: number of sub parts = 2 + 2 + 3 = 7



Case 4: If four lines are there: number of sub parts = 2 + 2 + 3 + 4 = 11



Case 5: If five lines are there: number of sub parts = 2 + 2 + 3 + 4 + 5 = 16



And in general, if there are n lines, number of sub parts

$$1+1+2+3+4+\ldots+n=1+\frac{n(n+1)}{2}=\frac{n^2+n+2}{2}$$